HW 4c: Pendulums and Chaos

1. Write a computer program that solves the damped, driven, nonlinear pendulum using the leapfrog algorithm. BTW in all plots asked for all parameters must be provided.
2. Construct the bifurcation diagram for the damped driven pendulum, how does it change with q?
3. How does the diagram change with the initial conditions?
4. Show a zoomed in portion of the bifurcation diagram to illustrate that it might be fractal in nature.
5. Challenge: fractals commonly have non-integer dimensions. Compute the dimension of the bifurcation diagram and show that it is greater than 1.
6. Provide a plot of angular displacement versus time as well as a phase space diagram for small driving force (overdamped).
7. Provide a plot of angular displacement versus time as well as a phase space diagram for a waveform that is period 2 (not initially but in late time behavior).
8. Construct a semilog plot for changing the I.C. and determine the Lyapunov exponent for a non-chaotic solution. How do you know that the motion is non-chaotic?
9. Repeat #8 for a chaotic solution.
10. Provide a plot of angular displacement versus time as well as a phase space diagram for chaotic motion.
11. Repeat on #10 using a slightly different I.C. illustrating how strikingly different the solution can be.
12. Comment on the differences between phase space diagrams for non-chaotic and chaotic motion.